SEMESTER 1

MATHEMATICS FOR ASASI UNIMAS

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PREFACE

Mathematics for Asasi UNIMAS Semester 1 is specially written for Asasi students taking Mathematics at Centre for Pre-University, Universiti Malaysia Sarawak and local matriculation centres.

The objective of this book is to provide a comprehensive discourse of the basic concepts and foundation in Mathematics. The extensive examples and guidelines for solving problems make the book more student-oriented. We have made every effort to make this book as effective, readable and attractive as possible.

All mathematical concepts are presented clearly in simple English for easy understanding. Worked examples are provided for every subtopic to show how typical mathematical problems are solved. Practice exercise, strategically placed throughout each chapter provide immediate practice to reinforce the concepts and skills taught at that point. Question and solution for each chapter provide a wide range of examination type question based on the concepts and theories learnt.

In each chapter, definition of concept, theorem, and formulae are highlighted for easy reference. Answers are also provided for all questions on each chapter. The book is also suitable for first year undergraduate Mathematics of a degree or diploma programme

We hope this book will assist Asasi students in mastering mathematical knowledge as well as preparing them for their examination.

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1

NUMBER SYSTEMS

1.1 Real Numbers

1. Natural numbers, $N = \{1, 2, 3, 4, ...\}$

2. Whole numbers, $W = \{0, 1, 2, 3, 4, ...\}$

3. Integers, $Z = \{ ..., -3, -2, -1, 0, 1, 2, 3, ...\}$

4. Negative integers, $Z^- = \{-1, -2, -3, ...\}$

5. Positive integers, $Z^+ = \{1, 2, 3, ...\}$

6. Rational numbers, $Q = \left\{ x : x = \frac{a}{b}, a, b \in Z, b \neq 0 \right\}$

7. Irrational numbers, $\overline{Q} = \left\{ ..., \pi, e, \sqrt{5}, \sqrt{2}, ... \right\}$

1.1.1 Real Number Line

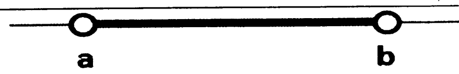
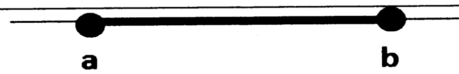
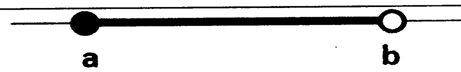
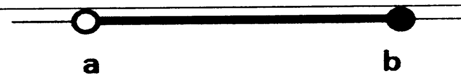
1. Intervals:

   Given two real numbers a and b, the possibilities are

   a. a is equal to b, $a=b$
   b. a is greater than b, $a>b$
   c. a is less than b, $a<b$

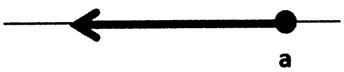
2. Finite intervals

If a and b are real numbers such that $a < b$, then the set of all real numbers x satisfying:

	Inequalities	Interval notation	Real number line
Open interval	$a < x < b$	(a,b)	
Closed interval	$a \le x \le b$	$[a,b]$	
Half-open or half-closed	$a \le x < b$	$[a,b)$	
Half-open or half-closed	$a < x \le b$	$(a,b]$	

3. Infinite intervals

If a is a real number, then the set of all real numbers x satisfying the conditions $x < a$, $x \le a$, $x > a$ or $x \ge a$ is called an infinite interval.

Inequalities	Interval notation	Real number line
$x < a$	(∞,a)	
$x \le a$	$(\infty,a]$	
$x > a$	(a,∞)	
$x \ge a$	$[a,\infty)$	

4. Union and intersection

a. The union of sets A and B denoted by $A \cup B$ is the set of all elements which belong to A or to B.

b. The intersection of two sets A and B denoted by $A \cap B$ is the set of elements which belong to both A and B.

Example 1.1

Define the following:
(a) Rational numbers.
(b) Irrational numbers.

Solution

(a) Rational numbers.

Rational numbers are set of numbers that can be written in fractions in the form $\frac{p}{q}$ where p and q are integers with $q \neq 0$, and their decimal representations are repeating or terminating. Rational numbers also can be define as $Q = \left\{ x : x = \frac{a}{b}, a, b \in Z, b \neq 0 \right\}$.

(b) Irrational numbers.

Irrational numbers are set of numbers that cannot be written in fractions and their decimal representations neither terminate nor recur.

Example 1.2

List any 2 rational numbers between 1 and 2.

Solution

$\frac{3}{2}, \ \frac{5}{3}$

Example 1.3

List any 2 irrational numbers between 3 and 4.

Solution

$\pi, \ \sqrt{11}$

Example 1.4

Justify the following as rational numbers:

(a) 1.232323...

(b) $0.\overline{387}$

(c) 0.7242424...

Solution

(a) 1.232323...

$$\begin{aligned} &\text{Let} & x &= 1.232323... & &\rightarrow (1) \\ &(1)\times 100; & 100x &= 123.232323... & &\rightarrow (2) \\ &(2)-(1); & 99x &= 122 \\ & & x &= \frac{122}{99} \end{aligned}$$

(b) $0.\overline{387}$

$$\begin{aligned} &\text{Let} & x &= 0.\overline{387} & &\rightarrow (1) \\ &(1)\times 1000; & 1000x &= 387.\overline{387} & &\rightarrow (2) \\ &(2)-(1); & 999x &= 387 \\ & & x &= \frac{387}{999} \\ & & x &= \frac{43}{111} \end{aligned}$$

(c) 0.7242424...

$$\begin{aligned} &\text{Let} & x &= 0.7242424... & &\rightarrow (1) \\ &(1)\times 10; & 10x &= 7.242424... & &\rightarrow (2) \\ &(1)\times 1000; & 1000x &= 724.242424... & &\rightarrow (3) \\ &(3)-(2); & 990x &= 717 \\ & & x &= \frac{717}{990} \end{aligned}$$

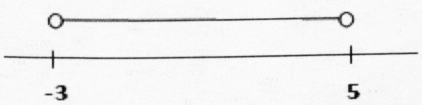
Example 1.5

Rewrite each of the following inequalities using interval notation and graph on the real number line:

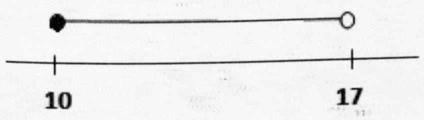
(a) $-3 < x < 5$
(b) $10 \le x < 17$
(c) $-\infty < x < -7$

Solution

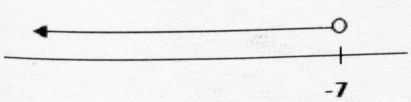
(a) $-3 < x < 5$
Interval notation: $(-3, 5)$



(b) $10 \le x < 17$
Interval notation: $[10, 17)$



(c) $-\infty < x < -7$
Interval notation: $(-\infty, -7)$



Example 1.6

Graph and solve each of the following :

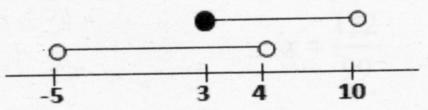
(a) $(-5,4)\cap[3,10)$

(b) $[3,12)\cup(8,\infty)$

(c) $(-\infty,5)\cap(2,3)$

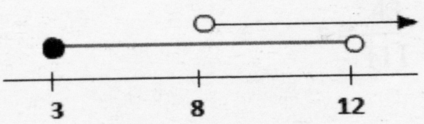
Solution

(a) $(-5,4)\cap[3,10)$



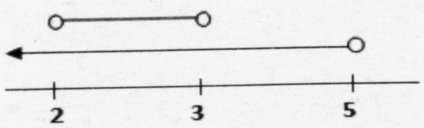
Answer : $[3,4)$

(b) $[3,12)\cup(8,\infty)$



Answer : $[3,\infty)$

(c) $(-\infty,5)\cap(2,3)$



Answer : $(2,3)$

1.2 Complex Numbers

1. The imaginary number $\sqrt{-1}$ is denoted by i.

2. Numbers of the form $a+bi$ where a and b are real numbers and $i=\sqrt{-1}$ are called complex numbers.

1.2.1 Algebraic Operations on Complex Numbers

1. Addition and subtraction of complex numbers: $(a+bi)\pm(c+di)=(a\pm c)+(b\pm d)i$.

2. Multiplication of complex numbers:
$$\begin{aligned}(a+bi)\times(c+di) &= ac+adi+bci+bdi^2\\ &= ac+adi+bci+bd\left(\sqrt{-1}\right)^2\\ &= ac+adi+bci-bd\\ &= (ac-bd)+(ad+bc)i\end{aligned}$$

3. Any pair of complex numbers in the form of $a\pm bi$ have a product which is a real number since $(a+bi)\times(a-bi)=a^2-abi+abi-b^2i^2=a^2+b^2$.

4. The complex number $(a+bi)$ and $(a-bi)$ are called conjugate numbers.

5. Division of complex numbers cannot be carried out because the denominator is made up of two independent terms. This can be overcome by multiplying the numerator and the denominator by the conjugate of the denominator

1.2.2 Algebraic Operations on Complex Numbers

If two complex numbers $a+bi$ and $c+di$ are equal, then $a=c$ and $b=d$.

Example 1.7

State the real and imaginary parts of the following complex number.

(a) $10+7i$

(b) $-2-\sqrt{3}\,i$

(c) $7i$

Solution

(a) $10+7i$

Let	$z=10+7i$
Real part,	$\text{Re}(z)=10$
Imaginary part,	$\text{Im}(z)=7$

(b) $-2-\sqrt{3}\,i$

Let	$z=-2-\sqrt{3}\,i$
Real part,	$\text{Re}(z)=-2$
Imaginary part,	$\text{Im}(z)=-\sqrt{3}$

(c) $7i$

Let	$z=7i$
Real part,	$\text{Re}(z)=0$
Imaginary part,	$\text{Im}(z)=7$

Example 1.8

Simplify:

(a) i^2

(b) i^3

(c) i^4

(d) i^{100}

Solution

(a) $i^2=-1$

(b)
$$\begin{aligned} i^3 &= i^2\cdot i \\ &= (-1)i \\ &= -i \end{aligned}$$

(c) $i^4 = i^2 \cdot i^2$

$= (-1)(-1)$

$= 1$

(d) $i^{100} = i^{2\times 50}$

$= (i^2)^{50}$

$= (-1)^{50}$

$= 1$

Example 1.9

Express each of the following in the form of $a+bi$:

(a) $(7+13i)+(2-8i)$
(b) $(3+5i)+8$
(c) $(6+8i)-(3-7i)$

Solution

(a)

$$(7+13i)+(2-8i) = (7+2)+(13-8)i$$

$$= 9+5i$$

(b)

$$(3+5i)+8 = (3+8)+5i$$

$$= 11+5i$$

(c)

$$(6+8i)-(3-7i) = (6-3)+(8+7)i$$

$$= 3+15i$$

Example 1.10

Simplify in the form of $a+bi$.

(a) $3(2+8i)$

(b) $(5+3i)i$

(c) $(2-5i)(4+2i)$

Solution

(a) $3(2+8i)=6+24i$

(b)
$$\begin{aligned}(5+3i)i &= 5i+3i^2\\ &= 5i+3(-1)\\ &= -3+5i\end{aligned}$$

(c)
$$\begin{aligned}(2-5i)(4+2i) &= 8+4i-20i-10i^2\\ &= 8-10(-1)-16i\\ &= 18-16i\end{aligned}$$

Example 1.11

Express the following in the form of $a+bi$.

(a) $\dfrac{1}{1+i}$

(b) $\dfrac{3}{4-3i}$

Solution

(a)
$$\begin{aligned}\frac{1}{1+i}\times\frac{1-i}{1-i} &= \frac{1-i}{1-i^2}\\ &= \frac{1-i}{1-(-1)}\\ &= \frac{1}{2}-\frac{1}{2}i\end{aligned}$$

(b)
$$\begin{aligned}\frac{3}{4-3i}\times\frac{4+3i}{4+3i} &= \frac{12+9i}{16-9i^2}\\ &= \frac{12+9i}{16-9(-1)}\\ &= \frac{12}{25}-\frac{9}{25}i\end{aligned}$$

Example 1.12

Find the square root of the following in the form of $a+bi$.

(a) $9+12i$

(b) $6-8i$

Solution

(a) Let $\sqrt{9+12i}=a+bi$

Squaring both sides,

$$\left(\sqrt{9+12i}\right)^2=(a+bi)^2$$

$$9+12i=a^2+2abi+b^2i^2$$

$$9+12i=a^2-b^2+2abi$$

Comparing,

$$a^2-b^2=9 \quad \rightarrow (1)$$

$$12i=2abi \quad \rightarrow (2)$$

$$ab=6$$

$$b=\frac{6}{a} \quad \rightarrow (3)$$

Substitute (3) into (1),

$$a^2-\left(\frac{6}{a}\right)^2=9$$

$$a^2-\frac{36}{a^2}=9$$

$$a^4-9a^2-36=0$$

$$\left(a^2-12\right)\left(a^2+3\right)=0$$

$$a^2=12 \quad \text{or} \quad a^2=-3 \text{ (reject, since } a \text{ must be real)}$$

$$a=\pm\sqrt{12}$$

Substitute values of a into (3).

$$b = \frac{6}{\sqrt{12}}$$

$$= \frac{6}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$$

$$= \frac{6\sqrt{12}}{12}$$

$$= \frac{\sqrt{4 \times 3}}{2}$$

$$= \frac{2\sqrt{3}}{2}$$

$$= \sqrt{3}$$

or

$$b = \frac{6}{-\sqrt{12}}$$

$$= \frac{6}{-\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$$

$$= \frac{6\sqrt{12}}{-12}$$

$$= \frac{\sqrt{4 \times 3}}{-2}$$

$$= \frac{2\sqrt{3}}{-2}$$

$$= -\sqrt{3}$$

Hence, $\sqrt{9+12i} = \pm\left(\sqrt{12} + \sqrt{3}\ i\right)$

(b) Let $\sqrt{6-8i} = a + bi$

Squaring both sides,

$$\left(\sqrt{6-8i}\right)^2 = (a+bi)^2$$

$$6 - 8i = a^2 + 2abi + b^2 i^2$$

$$6 - 8i = a^2 - b^2 + 2abi$$

Comparing,

$$a^2 - b^2 = 6 \quad \rightarrow (1)$$

$$-8i = 2abi \quad \rightarrow (2)$$

$$ab = -4$$

$$b = \frac{-4}{a} \quad \rightarrow (3)$$

Substitute (3) into (1),

$$a^2 - \left(\frac{-4}{a}\right)^2 = 6$$

$$a^2 - \frac{16}{a^2} = 6$$

$$a^4 - 6a^2 - 16 = 0$$

$$\left(a^2 - 8\right)\left(a^2 + 2\right) = 0$$

$$a^2 = 8 \quad \text{or} \quad a^2 = -2 \text{ (reject, since } a \text{ must be real)}$$

$$a = \pm\sqrt{8} = \pm 2\sqrt{2}$$

Substitute values of x into (3),

$$b = \frac{-4}{2\sqrt{2}}$$

$$= \frac{-4}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-2\sqrt{2}}{2}$$

$$= -\sqrt{2}$$

or

$$b = \frac{-4}{-2\sqrt{2}}$$

$$= \frac{4}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-2\sqrt{2}}{2}$$

$$= \sqrt{2}$$

Hence, $\sqrt{6-8i} = \pm\left(2\sqrt{2} - \sqrt{2}\, i\right)$

1.3 Indices, Surds and Logarithms Indices

1.3.1 Indices

If a is a real number and n is a positive integer than $a^n = a \times a \times a \times a \ldots \times a$ for n times. The number a is the base and n is called the index and a^n is read as a to the power of n.

1. Rules of Indices

   a. $a^m \times a^n = a^{m+n}$

   b. $a^m \div a^n = a^{m-n} \quad (m > n)$

   c. $(a^m)^n = a^{mn}$

   d. $(ab)^m = (a^m)(b^m)$

   e. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}, \quad b \neq 0$

2. Zero index
   $a^0 = 1, \quad (a \neq 0)$

3. Negative index
   $a^{-n} = \dfrac{1}{a^n}$

4. Rational index
   $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

5. Index property of equality

   For positive real number $a \quad (a \neq 1)$, if $a^x = a^y$, then $x = y$.

1.3.2 Surds

A surd is an irrational number of the form $\sqrt[n]{a}$, where n is a positive integer and a is a real number which is not a perfect square.

1. Algebraic operations on surds
   a. Addition and subtraction:
      Surds of the same kind can be combined by addition and subtraction,
      $5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$
   b. Multiplication (for all positive numbers a and b).
      $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
   c. Division (for all positive numbers a and b).
      $$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

2. Rationalizing the denominator
   (For all positive numbers a and b).
   $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$
   $\sqrt{a} - \sqrt{b}$ is the conjugate of $\sqrt{a} + \sqrt{b}$

   If a fraction has a surd in the denominator, the evaluation of the fraction is made easier by rationalizing the denominator of the fraction.

1.3.3 Logarithms

1. The logarithm of a positive number N to a given base a ($a>0$) is the power to which the base a must be raised in order to give the number: $a^x = N \Leftrightarrow x = \log_a N$

2. Specifically, $\log_a a = 1$, $\log_a 1 = 0$, $a^{\log_a x} = x$.

3. Logarithm of M to the base e is called the natural logarithm and is written as $\log_e x$ or $\ln x$ (pronounced as '*lon x*').

4. Logarithm of M to the base *10* is called the common logarithm and is written as $\log_{10} M$ *or* $\log M$ *or* $\lg M$

5. Laws of logarithms

a. $\log_a MN = \log_a M + \log_a N$

b. $\log_a \dfrac{M}{N} = \log_a M - \log_a N$

c. $\log_a M^p = p \log_a M$

d. $\log_a M = \dfrac{\log_b M}{\log_b a}$ and $\log_a b = \dfrac{1}{\log_b a}$

If x, y and a are positive real numbers such that $a \neq 1$, then $\log_a x = \log_a y \Leftrightarrow x = y$.

Example 1.13

Simplify the following:

(a) $2^3 \times 2^2$

(b) $(2y^4)(7y^2)$

(b) $(p^4 q^{-2})^2 (2p^{-2} q^4)^{-3}$

(c) $\dfrac{x^6}{x^2}$